Econ3101 - Section 006
Intermediate Microeconomics

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Due: Monday, April 23rd, 2012.

Homework 5.

The maximum score is 100 points. Present all nal answers neatly on these provided pages. Note that some Exercises have several parts, and each part may conceal more than one task for you. Be sure to answer every question thoroughly for full credit! Show any relevant calculations neatly. Please do your scratchwork somewhere else.
1. Consider a Cournot game with three rms. Each rm produces a homogeneous good and faces linear demand of the form \( P(Q) = a - bQ \), where \( Q = q_1 + q_2 + q_3 \), and a and b are positive constants as per usual. Assume that all firms face zero costs (i.e. \( C(q) = 0 \)).

(a) For each firm, derive the best response function below. Show all work.

(b) Solve for the Nash Equilibrium output of each rm. Show all work. You may not use the concept of symmetry to obtain your answer.
(c) Solve for the equilibrium price.

(d) Solve for the equilibrium profits of each firm.

2. Consider a Cournot game with a finite number, \( N \), of firms. Each firm produces a homogeneous good and faces linear demand of the form \( P(Q) = a - bQ \), where \( Q = q_1 + q_2 + \ldots + q_{N-1} + q_N = \sum_{i=1}^{N} q_i \), and \( a \) and \( b \) are positive constants as per usual. Assume that all firms face zero costs (i.e. \( C(q) = 0 \)).

(a) For each firm, derive the best response function below. Show all work. You may use the concept of symmetry to obtain your answer.
(b) Solve for the Nash Equilibrium output of each rm. Show all work. You may use the concept of symmetry to obtain your answer.

(c) Solve for the equilibrium price.

(d) Solve for the equilibrium profits of each firm.

(e) Now consider the limiting case. What is the Nash Equilibrium output of each firm when N is equal to infinity (i.e. \( \lim_{N \to \infty} q_i = ? \) ?)
3. Consider a Cournot duopoly with both firms producing a homogeneous good and facing linear demand of the form: \( P(Q) = 255 - 8Q \), where \( Q = q_1 + q_2 \). Assume the firms have the different costs. Firm 1 has costs: \( C_1(q_1) = 3q_1 + 2 \). Firm 2 has costs: \( C_2(q_2) = 4q_2 + 1 \).

(a) For each firm, derive the best response function below. Show all work.

(b) Graph the best response functions for both firms on the axes below. As in class, \( q_1 \) should be on the y-axis and \( q_2 \) should be on the x-axis.
(c) Solve for the Nash Equilibrium output of each firm. Show all work.

(d) Solve for the equilibrium price.

(e) Solve for the equilibrium profits of each firm.

4. Consider a Bertrand duopoly with both firms producing a homogeneous good and facing linear demand of the form: \( P(Q) = 255 - 8Q \), where \( Q = q_1 + q_2 \). Assume the rms have the different costs. Firm 1 has costs: \( C_1(q) = 3q \). Firm 2 has costs: \( C_2(q) = 4q \).

(a) For each firm, discuss the the best response strategy.
(b) Given this strategy, what is the Nash Equilibrium output of each firm?

(c) Solve for the equilibrium profits of each firm.

(d) What does this solution imply about the relationship between Bertrand Duopoly and perfect competition?
5. Consider a Stackelberg duopoly with both rms producing a homogeneous good and facing linear demand of the form: \( P(Q) = 255 - 8Q \), where \( Q = q_1 + q_2 \). Assume the rms have the different costs. Firm 1 has costs: \( C(q) = 3q + 2 \). Firm 2 has costs: \( C(q) = 4q + 1 \).

(a) Solve the profit maximization problem for firm 2. Circle your final answer.

(b) Given (a), what is the equilibrium output for each firm. Show all your work.

(c) Solve for the equilibrium price.

(d) Solve for the equilibrium profits of each firm.