Econ3101 - Section 006
Intermediate Microeconomics

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Due: Wednesday, February 1st, 2012.

Solutions to Homework 1.
1. A consumer has preferences for two goods. Her preferences satisfy Axioms 1 through 4 as discussed in class.

(a) Plot and label the following bundles:

- A (2,10)
- B (6,2)
- C (0,4)
- D (8,10)
- E (4,6)

(b) Assume A is indifferent to B ($A \sim B$). On a single line, list all the bundles in descending order of preference using ($\succ$) to denote strict preference and ($\sim$) to denote indifference between adjacent pairs. In other words, use the form:

$$A \succ B \succ C \succ D \succ E$$

Answer: $D \succ E \succ A \sim B \succ C$, or $D \succ E \succ B \sim A \succ C$. 


2. Consider an economic agent who has preferences that are represented by the utility function:

\[ u(x, y) = \sqrt{xy} \]

(a) for each pair of bundles A and B, indicate whether \( A \succ B \), \( A \prec B \), or \( A \sim B \) (but of course change the letters according to the bundles you are comparing).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>\quad</th>
<th>B \succ A</th>
<th>C \succ D</th>
<th>F \succ E</th>
<th>G \succ H</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(4,7)</td>
<td>B(7,10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(8,4)</td>
<td>D(2,8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E(7,3)</td>
<td>F(6,9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G(10,10)</td>
<td>H(9,6)</td>
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</tbody>
</table>

(b) Using the bundles in (a.), make a list that orders the bundles according to the agent’s preferences. Start with the most preferred bundle and end with the least preferred bundle using \( \succ \) or \( \sim \) (e.g. \( A \succ B \succ C \succ D \succ E \succ F \succ G \succ H \)).

**Answer:** \( G \succ B \succ F \sim H \succ A \succ E \succ D \)

(c) Consider a bundle \( C = (\frac{1}{2}, y) \), where C contains one-half of a unit of good x and some amount of a good y. If the consumer is indifferent between C and the bundle \( (1,1) \), how much of good y does the bundle C contain?

**Answer:**

\[ C \sim (1,1) \iff u\left(\frac{1}{2}, y\right) = u(1,1) \]

\[ \sqrt{\frac{1}{2}} \cdot y = \sqrt{1} \cdot 1 \]

\[ \frac{1}{2} \cdot y = 1 \]

\[ y = 2 \]
3. Consider the following four consumers/agents ($C_1, C_2, C_3, C_4$) with the following utility functions:

<table>
<thead>
<tr>
<th>Consumer</th>
<th>Utility function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$u(x, y) = 3x + 2y$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$u(x, y) = x^{\frac{1}{3}} y^{\frac{2}{3}}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$u(x, y) = \max(3x, y)$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>$u(x, y) = \min(2x, 2y)$</td>
</tr>
</tbody>
</table>

On the appropriate graph below, draw each consumer’s indifference curves through the following points:

$(2,2), (4,4), (6,6), (8,8)$ (*Note- this means you should draw an indifference through each of those points on each of the 4 graphs. The different graphs are for the different consumers/ utility functions.*)
$C_1$

$C_2$

$C_3$

$C_4$
4. A consumer’s preferences are given by the utility function:

\[ u(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}} \]

(a) Suppose the consumer owns a bundle \((x, y) = (12, 30)\). Label this bundle ”A” on the graph below and use red shading to indicate all the bundles that the consumer likes at least as much as A (e.g. bundles weakly preferred to A).

(b) Suppose the consumer owns a bundle \((x, y) = (8, 25)\). Label this bundle ”B” on the graph below and use black shading to indicate all the bundles that the consumer does not like at least as much as B (e.g. B is weakly preferred to these bundles).

(c) Calculate and show your derivations below for the marginal utility of X and of Y for the utility function given above.

\[ MU_x = \frac{1}{2} x^{-\frac{1}{2}} \cdot y^{\frac{1}{2}} \]

\[ MU_y = \frac{1}{2} x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \]
5. Suppose there are two agents, Jeremy Bentham and John Stuart Mill (look them up if you want some bedtime reading), who tell you the following about their preferences:

- For Jeremy Bentham, the marginal utility of X is always equal to -25 utils, and the marginal utility of Y is always equal to 40 utils.

- For John Stuart Mill, the marginal utility of X is always equal to -5 utils, and the marginal utility of Y is always equal to -15 utils.

In the appropriate graph below, sketch a few of the person’s indifference curves and indicate the direction at which their preferences are increasing.

Figure 1: Jeremy Bentham

Figure 2: John Stuart Mill
6. I’ve been told I have odd tastes. I thought about this and realized that:

- If I have more Y than X, then I always trade 5 Y for one X.
- If I have more X than Y, then every time I give up one Y, I require 5 X to remain indifferent after the exchange.

(a) Plot and label the following bundles and then graph my indifference curves through each of these points.

\[
\begin{align*}
A &= (10,50) \\
B &= (50,50) \\
C &= (65,5)
\end{align*}
\]

(b) Please tell me the value of my:

\[
\begin{align*}
\text{Marginal Rate of Substitution at point } A &= 5 \\
\text{Marginal Rate of Substitution at point } C &= 1/5
\end{align*}
\]
7. Britney S. has a divided mind. Her preferences change drastically according to how much X and Y she has. I have sketched some of her indifference curves below, where arrows indicate the direction of increasing preferences notice how Britneys direction of preferences changes depending on which quadrant we consider. Characterize the properties of the quadrants by filling in the blanks in the following statements.

Within Quadrant III,

- The $MU_x$ is (positive/negative): negative.
- The $MU_y$ is (positive/negative): negative.
- The preferences (do/don’t) don’t satisfy the axiom of non-satiation.
- The preferences (do/don’t) don’t satisfy the axiom of convexity.
Within Quadrant IV,

- The $MU_x$ is (positive/negative): **positive**.
- The $MU_y$ is (positive/negative): **negative**.
- The preferences (do/don’t) **don’t** satisfy the axiom of non-satiation.
- The preferences (do/don’t) **don’t** satisfy the axiom of convexity.

Within Quadrant I,

- The $MU_x$ is (positive/negative): **positive**.
- The $MU_y$ is (positive/negative): **positive**.
- The preferences (do/don’t) **do** satisfy the axiom of non-satiation.
- The preferences (do/don’t) **don’t** satisfy the axiom of convexity.

Within Quadrant II,

- The $MU_x$ is (positive/negative): **positive**.
- The $MU_y$ is (positive/negative): **negative**.
- The preferences (do/don’t) **don’t** satisfy the axiom of non-satiation.
- The preferences (do/don’t) **do** satisfy the axiom of convexity.