Midterm 2 will be on Wednesday, April 4th, 2012.

Practice Midterm 2.

Included are a few practice problems to help you prepare for the second midterm exam.

As for the exam, calculators are allowed, rulers are allowed, and the use of pencil as opposed to pen is strongly encouraged.

Good luck studying!
1. Consider a Cournot duopoly with both firms producing a homogeneous good and facing linear demand of the form: \( P(Q) = 50 - 30Q \), where \( Q = q_1 + q_2 \). Assume both firms have the same costs: \( C(q) = 6q \).

(a) For each firm, derive the best response function below. Show all work.

(b) Graph the best response functions for both firms on the axes below. Indicate the axes intercepts for both functions.
(c) Solve for the Nash Equilibrium output of each firm. Show all work.

(d) Compute the equilibrium market price.

(e) Compute the equilibrium profits of each firm.

(f) Assume a Stackelberg setting instead, where firm 1 moves first. How does profit maximization for each firm differ with respect to the Cournot case? Solve for the new Nash Equilibrium quantities, price and profits.
2. Intel and AMD produce computer processors (a homogeneous product). Market demand is given by: \( P(Q) = 96 - 10Q \), where \( Q = q_I + q_A \), \( I \equiv \) Intel, and \( A \equiv \) AMD. Neither firm has any costs (e.g. \( C(q) = 0 \)).

The problem is, they can’t decide how to behave toward one another. One day they want to collude and act like a joint monopoly; the next day they are at each other like Cournot competitors; then they think they are Bertrand competitors; and by the end of the week Intel pushes to the front and acts like a Stackelberg leader. Then it just starts all over again.

Complete the following table, filling in equilibrium values of the respective variables under the different sorts of competition between these two firms. Wherever applicable, instead of expressing your answer as a decimal, express your answer as a fraction.

<table>
<thead>
<tr>
<th>Competition</th>
<th>( q_I^* )</th>
<th>( q_A^* )</th>
<th>( q_I^* + q_A^* )</th>
<th>( P(Q) )</th>
<th>( \pi_I )</th>
<th>( \pi_A )</th>
<th>( \pi_A + \pi_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint Monopoly</td>
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<tr>
<td>Cournot</td>
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<tr>
<td>Bertrand</td>
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<tr>
<td>Stackelberg</td>
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</tbody>
</table>
3. Consider two players: Alice and Bob. Alice moves first. At the start of the game, Alice has two piles of coins in front of her: one pile contains 4 coins and the other pile contains 2 coins. Each player has two moves available: either “take” the larger pile of coins and give the smaller pile to the other player or “push” both piles across the table to the other player. Each time the piles of coins pass across the table, the quantity of coins in each pile doubles. The game lasts for a maximum of 4 rounds (two choices each), in which case they enter a lottery. (They both just care about their expected payoff, they’re ”risk neutral”).

- Alice has 3/4 probability of winning while Bob only has 1/4, and the winner takes all. What is the subgame perfect Nash Equilibrium of this game?

- Now Alice owns all the tickets to the lottery, would she give any to Bob for free before playing the entire game? If so, what fraction would she give him?. Why?

4. Suppose now, Alice and Bob make it past the 4 rounds hence earning 96 coins between the two of them. However, instead of splitting it evenly, they play another game. Bob moves first. He offers a fraction of the prize sum to Alice, then she decides whether to accept or reject. If she rejects, they lose half of the pot and then she gets to make an offer over the coins left. Bob accepts or rejects. If he rejects, the pot is again reduced by 50% and they enter a lottery.

- Alice has 3/4 probability of winning while Bob only has 1/4, and the winner takes all. What is the subgame perfect Nash Equilibrium of this game?

- Now Alice owns all the tickets to the lottery, would she give any to Bob for free before playing the entire game? If so, what fraction would she give him?. Why?
5. Consider a Perfectly Competitive industry where each (identical) firm has the following total cost function:

\[ C(q) = 1 + 81q^2 \]

The market demand facing the industry is:

\[ Q^d(p) = (120 - p)^{\frac{1}{9}} \]

where \( p \) is the market price

(a) If there are currently 72 firms in the industry, for each firm what is the short-run equilibrium market price \((p^*)\), quantity \((q^*)\), and profits \((\pi^*)\). Show your work.

(b) **Long Run**

i. For each firm, what are the long-run equilibrium market price \((p^*)\), quantity \((q^*)\), profits \((\pi^*)\), and number of firms in the industry \((N^*)\). Show your work.

6. Now suppose that the government levies an Excise (per unit) Tax \( t = 1.25 = \frac{5}{4} \) on all firms in the economy. Use the demand and cost structure from the previous exercise.
(a) **Short Run:** Suppose that the number of firms in the industry is $N = 72$ again, what are the short-run equilibrium market price ($p^*$), quantity ($q^*$), and profits ($\pi^*$) for each individual firm. Show your work.

(b) **Long Run:** For each firm, what are the long-run equilibrium market price ($p^*$), quantity ($q^*$), profits ($\pi^*$), and number of firms in the industry ($N^*$). Show your work.

(c) How much does the government collect in taxes from each firm? Assume the government adds a lump sum tax rebate equal to that amount in addition to the excise tax. Repeat whatever steps you need to find the equilibrium both in the short run and in the long run under this new policy. (Use latest $N^*$ found).
7. Patents and copyrights can be thought of the right to have monopolistic power over your own invention/creation. However, one must pay a fee in order for the petition to be considered, and if conditions are met, for the right to be granted. Assume the government consults a psychic and is able to anticipate an inventor’s creation. The psychic is very specific and tells the government information about the cost function and the demand for that good. Assume the government acts only in the citizens/consumers’ best interests (the biggest of all assumptions), and decides to modify the incentives so that the monopolist chooses to produce the optimal quantity at the lowest cost possible to taxpayers. What excise subsidy and patent fee should the government set? (The fixed cost of developing the invention has already been paid hence it is not necessary to take it into account).

\[ C(q) = \text{Fee} + 3q \]
\[ P(q) = 50 - (1/6)q \]