Econ3101 - Section 006
Intermediate Microeconomics

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Midterm 1 will be on Wednesday, February 22nd, 2012.

Practice Midterm 1.

(*note: this list of exercises is not exhaustive and it may be the case that I ask about topics not covered here but seen in class.)
1. **Theory of Choice**

Consider a consumer who has the following utility function: \( u(x, y) = 3x + 2y \)

(a) Draw the indifference curve through the bundle \((x_0, y_0) = (10, 10)\), and indicate the direction of preferences.

(b) Do these preferences satisfy convexity (Axiom 4)? Justify your answer. You may use the above graph to illustrate your answer. If the preferences do in fact satisfy Axiom 4, relate your answer to the law of diminishing marginal utility.

(c) Do these preferences satisfy non-satiation (Axiom 3)? Justify your answer.
2. Consumer Optimization and Demand A

Consider a consumer who has the utility function: \( u(x, y) = 30x^{\frac{2}{3}}y^{\frac{1}{3}} \).

(a) For \( P_x > 0, P_y > 0 \) and \( M > 0 \), can one find an interior solution (e.g. \( x^* > 0 \) and \( y^* > 0 \)) to this consumer’s maximization problem? Justify your answer.

(b) What is the direction of preferences for this consumer? Justify your answer. Also, indicate the direction on the graph below.

(c) Given \( P_x = 10, P_y = 20 \) and \( M = 200 \), draw the budget constraint.
3. Consumer Optimization and Demand B

Again, consider a consumer who has the utility function: \( u(x, y) = 30x^\frac{2}{3}y^\frac{1}{3} \).

(a) Write down the consumer’s maximization problem.

(b) Find the demand functions \( x^*(P_x, P_y, M) \) and \( y^*(P_x, P_y, M) \) that solve the consumer’s maximization problem.

(c) Given \( P_x = 10, P_y = 20 \) and \( M = 200 \) as before, how much of goods x and y does the consumer consumer? Draw and label \((x^*, y^*)\) on the graph in (2c).
4. Consumer Optimization and Demand C
Again, consider a consumer who has the utility function: \( u(x, y) = 30x^{\frac{2}{3}}y^{\frac{1}{3}} \). The following questions use the demand functions you derived in (3c).

(a) Given \( P_y = 20 \) and \( M = 200 \), draw the consumer’s demand for good \( x \) as a function of \( P_x \).

(b) What fraction of consumer’s income does she spend on good \( x \)? What fraction does she spend on good \( y \)? Do \( S_x \) and \( S_y \) depends on the consumer’s prices or income? Given these preferences, is there any interesting intuition for this dependence (or lack thereof) of income shares on prices and/or income?

(c) Determine whether goods \( x \) and \( y \) are substitutes, complements or are unrelated (inelastic) goods. Is there any interesting intuition for your result?

(d) Is good \( x \) a Giffen good? How about good \( y \)?
5. Consumer Optimization and Demand D

Again, consider a consumer who has the utility function: \( u(x, y) = 30x^{\frac{2}{3}}y^{\frac{1}{3}} \). The following questions use the demand functions you derived in (3c).

(a) Determine whether goods \( x \) and \( y \) are inferior or normal goods.

(b) Calculate the consumer’s income elasticities \( \eta_x \) and \( \eta_y \). How would the consumer’s demand for good \( x \) change if her income doubled? How would her demand for \( x \) change if her income halved? How would the consumer’s demand for good \( y \) change if her income doubled? How would her demand for \( y \) change if her income halved?

6. Consumer Optimization A to D

In questions 2 to 5 I gave you a particular utility function to work with. In principle I could have asked you to use many different types of utility function. You may find it useful to redo the exercises using the following utility functions:

<table>
<thead>
<tr>
<th>Type</th>
<th>Function</th>
<th>Exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobb-Douglas</td>
<td>( u(x, y) = Ax^{\alpha}y^{\beta} ), where ( A, \alpha, \beta &gt; 0 )</td>
<td>Problem 2 to 5</td>
</tr>
<tr>
<td>Stone-Geary</td>
<td>( u(x, y) = (x + a)(y + b) ), where ( a, b &gt; 0 )</td>
<td>Problem 2 to 5</td>
</tr>
<tr>
<td>Leontief</td>
<td>( u(x, y) = \min(ax, yb) ), where ( a, b &gt; 0 )</td>
<td>Problem 2 to 5 and 8</td>
</tr>
<tr>
<td>Linear</td>
<td>( u(x, y) = ax + by ), where ( a, b &gt; 0 )</td>
<td>Problem 2 to 4</td>
</tr>
</tbody>
</table>

If you would like more practice yet, I encourage you to consider a new utility function, **quasi-linear utility**: \( u(x, y) = x^{\frac{1}{2}} + y \). A hint to get you started is that this utility function will satisfy all of the axioms and therefore you can use calculus to derive the tangency condition.
7. Consumer Optimization and Demand E
Consider a consumer who has the utility function: \( u(x, y) = (20 + x)(10 + y) \)
At any point \((x, y)\), the marginal utility of good \( x \) and the marginal utility of good \( y \) are:

\[ MU_x(x, y) = \]

\[ MU_y(x, y) = \]

(a) If the price of good X is 8 dollars, and the price of good Y is 50 dollars, how much of good X will consumer buy when her income is 320 dollars?

(b) If the price of good X is 8 dollars, and the price of good Y is 5 dollars, how much of good X will consumer buy when her income is 320 dollars?

(c) If the price of good X is 20 dollars, and the price of good Y is 5 dollars, how much of good X will consumer buy when her income is 320 dollars?
8. **Labor Analysis**

Consider a consumer/worker who consumes food \((C)\) at price \(p\) and leisure \((l)\) at price/wage \(w\). Suppose that the government has imposed a mandatory maximum workday of 16 hours \((\bar{L} = 16)\). Lastly, suppose that consumer/worker has preferences: 
\[ u(l, C) = \min(l, C). \]

(a) Derive the budget constraint as done in class.

(b) Derive the demand functions for food \((C^*)\) and leisure \((l^*)\).

(c) How do consumption and leisure \((C^*, l^*)\) change with a change in the maximum workday \((\bar{L})\)? Interpret your result.

(d) How do consumption and leisure \((C^*, l^*)\) change with a change in the price of food \((p)\)? Interpret your result.

(e) How do consumption and leisure \((C^*, l^*)\) change with a change in the price of leisure \((w)\)? Interpret your result.
9. **Intertemporal Choice**
Suppose you have a consumer with endowment/income $M$ which must be consumed or saved in period 1. The amount saved ($s$) will earn an additional interest rate of $r$ and can be consumed in period 2. The agent has preferences $u(x, y) = \log(c_1) + \beta \log(c_2)$.

(*Note: $\frac{d}{dx} \log(x) = \frac{1}{x}$ and $\frac{d}{dx} \beta \log(x) = \frac{\beta}{x}$) (*Show your work!)

(a) What is the budget constraint for period 1?

(b) What is the budget constraint for period 2?

(c) Now, combine the two budget constraints into one and solve. What is the consumer’s demand function for period 1 ($c_1^*$)?

(d) What is the consumer’s demand function for period 2 ($c_2^*$)?
10. **Returns to Scale**

Recall the definition of Returns to Scale (e.g. for all $\lambda > 1$). For the following production functions, justify whether the production function is Constant, Increasing or Decreasing Returns to Scale. All credit for the answer will be based on your justification / proof. That is, without the proper justification no credit will be given to answering a correct response to the type of returns to scale.

**Cobb-Douglas**

$$F(K, L) = K^{1/4}L^{1/8}$$

$$F(K, L) = 50K^{1/2}L^{1/2}$$

$$F(K, L) = 5K^3L$$

**Constant elasticity of substitution (CES)**

$$F(K, L) = \left(K^{1/5}L^{1/5}\right)^5$$

**Linear**

$$F(K, L) = 4K + 3L$$

**Separable Form**

$$F(K, L) = 7\sqrt{K} + 2\sqrt{L}$$

$$F(K, L) = 2K^3 + 4L^3$$
11. **Cost Minimization 1**

Suppose the production function of a firm takes the following CES form:

\[ F(K, L) = (K^{1/3} + L^{1/3})^3 \]

(a) Consider the firm’s operation in the short run. Capital is fixed at \( K = 100 \). The firm faces factor prices \( w = 1 \) and \( r = 7 \). Derive the equations for short-run total cost, average total cost and marginal cost. Show all work.

\[ STC(Q) = \]

\[ SAC(Q) = \]

\[ SMC(Q) = \]

(b) Consider the firm’s operation in the long run. The firm is free to choose any level of capital, as it is now fully exible. The firm is also free to choose any level of labor, as it did in the short run. First derive the marginal products of capital and labor.

\[ MPL(L, K) = \]

\[ MPK(L, K) = \]

\[ MRTS(L, K) \]

Derive the equations for long-run total cost, average total cost, and marginal cost when factor prices are \( w = 1 \) and \( r = 7 \). Show all work.

\[ LTC(Q) = \]

\[ LAC(Q) = \]

\[ LMC(Q) = \]
12. Suppose the production function of a firm takes the following Leontief form:

\[ F(L, K) = \min(40K, 20L) \]

(a) Sketch a Q-level isoquant below. The y-axis should be labelled with K and x-axis with L. Identify the coordinates of the kink of the isoquant.

(b) Suppose that the firm faces factor prices \( w = 50 \) and \( r = 50 \). Derive its long run total cost function, \( LTC(Q) \). Hint: I am looking for the exact equation for the \( LTC(Q) \), as a function of \( Q \). You should use the graph of the isoquant to help you obtain the solution.
13. Suppose the production function of a firm takes the following Linear form:

\[ F(L, K) = 5K + 3L \]

(a) Below, graph the firm’s isoquant when \( Q = 75 \). Use black ink.

(b) Graph the isocost curve that the firm would choose when \( w = 3 \) and \( r = 2 \). Use pencil for this curve. Below, report the optimal combination of capital and labor for these factor prices and report the cost to the producer.

\((L^*, K^*) = \)

\(LTC(Q) = \)
(c) Graph the isocost curve that the firm would choose when \( w = 1 \) and \( r = 5 \). Use blue ink for this curve. Below, report the optimal combination of capital and labor for these factor prices and report the cost to the producer.

\[
(L^*, K^*) =
\]

\[LTC(Q) = \]

(d) Graph the isocost curve that the firm would choose when \( w = 6 \) and \( r = 10 \). Use red ink for this curve.

14. **Profit maximization 1.**

Consider a perfectly competitive industry where each (identical) firm has the following total cost function:

\[
C(q) = 1 + 225q^2
\]

Find the supply function in the long run and in the short run.
15. **Cost Minimization 2**

Suppose the production function of a firm takes the following Cobb-Douglas form:

\[ F(K, L) = 3K^{1/3}L^{2/3} \]

(a) Consider the firm’s operation in the short run. Capital is fixed at \( K = 100 \). The firm faces factor prices \( w = 1 \) and \( r = 7 \). Derive the equations for short-run total cost, average total cost and marginal cost. Show all work.

\[ STC(Q) = \]

\[ SAC(Q) = \]

\[ SMC(Q) = \]

(b) Consider the firm’s operation in the long run. The firm is free to choose any level of capital, as it is now fully flexible. The firm is also free to choose any level of labor, as it did in the short run. First derive the marginal products of capital and labor.

\[ MPL(L, K) = \]

\[ MPK(L, K) = \]

\[ MRTS(L, K) \]

Derive the equations for long-run total cost, average total cost, and marginal cost when factor prices are \( w = 1 \) and \( r = 7 \). Show all work.

\[ LTC(Q) = \]

\[ LAC(Q) = \]

\[ LMC(Q) = \]
16. **Profit maximization 2.**
   Find the supply function in the long run of a firm whose technology is represented by the production function in the previous exercise. Check all conditions.

Is the supply function in the short run different?

What is the quantity supplied in the long run when \( p = 10, \ w = 1 \) and \( r = 7 \)?