The AK Growth Model

Endogenous Growth: A Brute Force Approach

- The reason there is no long-run growth without TFP growth in the Solow model is because of diminishing marginal returns to capital assumed in Inada conditions.
- One can see that as long as \( \lim_{k \to \infty} sf'(k) < n + d \) the savings curve will eventually cross the required investment line and there will be a steady state.
- Alternatively, look at the capital dynamics in the Solow model:
  \[
  \frac{K}{K} = sAK^{\alpha-1} - (n + d)
  \]
- So, capital growth goes to zero unless \( \alpha = 1 \)
- Setting \( \alpha = 1 \) gives \( Y = AK \), which is the AK model.
Solow Diagram for different $\alpha$ values

The AK model
**Endogenous Growth: A Brute Force Approach**

- Use the capital accumulation equation:
  \[ \frac{\dot{K}}{K} = s \frac{Y}{K} - d = sA - d \]

- Key points to note:
  - The economy is always on the balanced growth path (whereas in the Solow model we can only talk about BGP for an economy that has completed the transition)
  - Savings rate does affect the long-run growth rate
  - An important shortcoming of this simple AK model is that when \( \alpha = 1 \), capital is the only factor of production, violating one of Kaldor’s facts.

**Growth through Externalities**

- Romer (1986) and Lucas (1988) proposed two different models that end up looking very similar to the AK model.
- Both models manage to circumvent the problem with increasing returns (e.g., firms would like to produce an infinite amount.)
  - Cobb-Douglas production for each firm: \( Y = AK^\alpha L^{1-\alpha} \)
  - Key assumption: \( A \) is determined at the aggregate level as \( A = \bar{A}K^{\bar{\alpha}} \) whereas each firm takes \( A \) as given.
  - In other words, firms ignore that the capital they accumulate at the aggregate level is a determinant of productivity. Therefore, they create a positive externality on other firms.
  - Aggregate production function is:
    \[ Y = AK^\alpha L^{1-\alpha} = (\bar{A}K^{\bar{\alpha}})K^\alpha L^{1-\alpha} = \bar{A}KL^{1-\alpha} \]
The AK model and Policy Debates

The fact that savings rate can affect the growth rate (and in a big way) made the AK model very popular in policy discussions.

It makes government policy potentially very important for growth.

In a famous paper, Lucas (1990) called tax cuts on savings as the “largest genuinely free lunch I have seen in 25 years in this business.”

Even today when candidates fiercely debate tax policy, an important part of discussion revolves around growth.

King and Rebelo (1990, JPE): The “welfare effect” of a 10 percent increase in income tax is 40 times larger in an (AK) endogenous growth model (65% of consumption) than it is in a neoclassical growth model (1.6% of consumption)

Stokey and Rebelo (1995) and Lucas (1990) argue that if endogenous growth models are calibrated to plausible values the effect on welfare is not likely to be large.

Note that this “gift” of the AK model is also its “curse.”

Because if tax differences are so important for growth, how come countries like Sweden with extremely high tax rates grow as fast as the US?
Shortcomings of the AK model

- Growth is the outcome of accidents—actions that are completely unintentional.
- Externalities must be substantial: For example, the capital bought by an investor contributes twice as much to others’ production than to his/her own. Same for human capital: Your education benefits others more than it benefits you.
- Alternatively stated, the Social return on many types of investments far exceed their private return.
- If externalities are really that big, individuals will typically find a way to capitalize on them (a doctor will not distribute advice on the street, etc.)
- Coefficient on externality must be exactly 1-α. Otherwise, there will be no BGP