Say a little bit about De Loecker and Warzynski (AER 2012)

- Brings together Hall (1988) approach to marginal cost estimation with Olley-Pakes (1996) approach to production function estimation.

- $Q_{ft} = F_t(V_{ft,}K_{ft})\Omega_{ft}$

  - firm f, at time t

  - V_{ft} variable inputs, K_{ft} fixed inputs facing adjustment costs

  - W^v_{ft} and W^k_{ft} price of variable and fixed inputs

- Note Assumptions

- F_t twice continuously differentiable...

- Hicks Neutral productivity is log additive and firm specific

- Firms minmizes short-run costs taking output quantity and input prices are agiven.

Set up Lagrangian (leave ft implicit)

$$\begin{aligned} L(V,K,\lambda) &= \sum_{v=1}^{V} W^v V^f + \sum_{k=1}^{K} W^k K^k \\ &\quad + \lambda \left[Q - F(V,K,\Omega) \right] \end{aligned}$$

The FONC

$$\frac{\partial L}{\partial V^v} = W^v - \lambda \frac{\partial F}{\partial V^v} = 0$$

- λ_{ft} is the marginal cost to firm f. Rearranging and mult-plying both sides by $\frac{V}{Q}$ yields

$$\frac{\partial F}{\partial V^v}\frac{1}{W^v}\frac{V}{Q} = \frac{1}{\lambda}\frac{V}{Q}$$

$$\frac{\partial F}{\partial V^v}\frac{V}{Q} = \frac{1}{\lambda}\frac{W^v V}{Q}$$

$$= \frac{P}{\lambda}\frac{W^v V}{PQ}$$

- Wanted

$$\begin{aligned}
\textit{marginal cost} &: \lambda \\
markup &: \mu = \frac{P}{\lambda} \\
output\ elasticity &: \theta^v = \frac{\partial F}{\partial V^v}\frac{V}{Q} \\
\text{expenditure share} &: \alpha^v = \frac{W^v V}{PQ}
\end{aligned}$$

because

$$\theta^v = \mu\alpha^v$$

- Write output a single-product firm f at time t in logs

$$
\begin{aligned}
q_{ft} &= f(x_{ft}, ; \beta) + \omega_{ft} + \varepsilon_{ft} \\
&= f(v_{ft}, k_{ft}; \beta) + \omega_{ft} + \varepsilon_{ft}
\end{aligned}
$$

- For
  - variable inputs $v_{f,t}$ and fixed input k_{ft}.
  - ω_{ft} structural error
  - ε_{ft} forecast error or measurement error.

- If have β can calculate θ_t^v. There are econometric issues in estimating β. Follow Levinsohn and Petrin (2003) and Olley and Pakes and use a control function approach.

– Let $m_{ft} = m_t(k_{ft}, \omega_{ft}, z_{ft})$ be demand for materials (z_{ft} include things like input pricess)

– Monotonic under mild conditions, so invert it, to $\omega_{ft} = h_t(m_{ft}, k_{ft}, z_{ft})$, so have

$$q_{ft} = f(v_{ft}, k_{ft}; \beta) + h_t(m_{ft}, k_{ft}, z_{ft}), + \varepsilon_{ft}$$

- Let's use the translog functional form for $f(v_{ft}, k_{ft}; \beta)$, and focus on a value-added version,

$$q_{ft} = \beta_l l_{ft} + \beta_k k_{ft} + \beta_{ll} l_{ft}^2 + \beta_{kk} k_{ft}^2 + \beta_{lk} l_{ft} k_{ft} + \omega_{ft} + \varepsilon_{ft}.$$

- In the first stage run the following nonparametric regression:

$$q_{ft} = \phi_t(l_{ft}, k_{ft}, m_{ft}, z_{ft}) + \varepsilon_{ft}$$

and obtain estimates of expected output $\hat{\phi}_{ft}$ and an estimate for $\varepsilon_{ft.}$. Expected output given by

$$\phi_{ft} = \beta_l l_{ft} + \beta_k k_{ft} + \beta_{ll} l_{ft}^2 + \beta_{kk} k_{ft}^2 + \beta_{lk} l_{ft} k_{ft} + h_t(m_{ft}, k_{ft}, z_{ft})$$

- Note for given values of β can solve out for estimate of $\hat{\omega}_{ft}$

- Second stage uses law of motion for productivity

$$\omega_{ft} = g_t(\omega_{ft-1}) + \xi_{ft}$$

- Orthgonality condition is that the innovation ξ_{ft} is uncorrelated with $t-1$ information.

– For a given β run nonparametric regression of $\hat{\omega}_{ft}$ on $\hat{\omega}_{ft-1}$ and then construct $\xi_{ft}(\beta)$. Then form moments

$$E\left(\xi_{ft}(\beta)\begin{pmatrix} l_{ft-1} \\ k_{ft} \\ k_{ft}^2 \\ l_{ft-1}k_{ft} \end{pmatrix}\right) = 0$$

- Once have estimate of β, off to the races...