Econ 8602—Spring 2024—Take-Home Final

Please work alone. You can choose when to work on it, but please work only one day on it.

Do three out of the following four questions:

Question 1. Perfect Price Discrimination. Consider the Logit Model of Product Differentiation from homework 1. There are n firms indexed by j and an "outside good" labeled by 0. Each firm $j \geq 1$ has constant marginal cost equal to c_j. There is a unit measure consumers. Let i index an individual consumer and suppose the utility of consumer i from purchasing good j and paying price $p_{i,j}$ has utility

$$\begin{aligned} U_{i,j} &= \xi_j - \alpha p_{i,j} + \varepsilon_{i,j} \text{ for } j = 1, 2, ...n \\ &= \varepsilon_{i,0} \text{ for good 0.} \end{aligned}$$

A consumer is therefore summarized by his or her vector of draws $\boldsymbol{\varepsilon}_i = (\varepsilon_{i,0}, \varepsilon_{i,1}, ...\varepsilon_{i,n})$ since consumers are otherwise the same. Assume the $\varepsilon_{i,j}$ are drawn type 1 extreme value which delivers the logit choice probabilities.

We allow for perfect price discrimination. Each firm observes the entire vector of draws $\boldsymbol{\varepsilon}_i$ for each consumer i and can set prices contingent on $\boldsymbol{\varepsilon}_i$ (i.e. can set a price $p_{i,j}$ specific to individual i.). The n firms compete in a Bertrand fashion for each individual consumer.

(a) Take as given the n firms in the industry and calculate the equilibrium of price competition when perfect price discrimination is feasible. Derive formulas for the market shares of each firm.

(b) A potential entrant is considering entry into this industry. If it comes in, it will be firm $j = n + 1$, and consumers will all get a new logit draw $\varepsilon_{i,n+1}$ for this firm. The firm has given values of ξ_{n+1} and c_{n+1}. If the firm enters, it pays a fixed cost ϕ_{n+1}. Derive a condition determining whether the firm enters. Discuss the connection between the private incentive for entry and the social incentive. That is, how does the total of producers and consumers surplus change if the firm enters?

Question 2

Consider a model of the shipbuilding industry. At the beginning of period t there is a global stock of q_t° vessels. There are two shipbuilders indexed by $j \in \{1, 2\}$. In period t, each shipbuilder j decides whether to produce one unit of a ship, denoted $x_{jt} = 1$, or no ship, $x_{jt} = 0$. After these production decisions, the end-of-period total stock of identical vessels is

$$\hat{q}_t = q_t^\circ + x_{1t} + x_{2t}.$$

There is a rental market for ships and the per-period rent in period t depends upon the end-of-period stock $\hat{q}_t$, according to the rental demand function $r_t = D(\hat{q}_t)$, which is strictly downward sloping.

The sale price of a ship at the end of period t equals

$$p_t(\hat{q}_t) = r_t + E\left[\beta r_{t+1} + \beta^2 r_{t+2} + \beta^3 r_{r+3} + ...|\hat{q}_t\right]$$

This equals the present value of rents, conditioned on global vessel quantity $\hat{q}_t$. Note that the market sees the realized values of x_{1t} and x_{2t} and these get factored into the sale price of a ship in period t. The revenue of shipbuilding firm j in period t, given $\hat{q}_t$ and x_{jt}, is

$$rev_{jt} = p_t(\hat{q}_t)x_{jt}$$

The period t profit of firm 1 depends upon whether it produces a ship in the period of not. The profit is

$$\begin{aligned} \pi_{jt}^0 &= \varepsilon_{jt}^0, \text{ if } x_{jt} = 0, \\ \pi_{jt}^1 &= E\left[rev_{ij}\right] - \kappa_j + \varepsilon_{jt}^1, \text{ if } x_{jt} = 1, \end{aligned}$$

where the ε_{jt}^0 and ε_{jt}^1 are i.i.d. drawn from the standard type-one extreme value distribution. Note that the marginal cost κ_j potentially differs by j.

When each firm chooses whether or not to produce, firm j sees its own draws of the profit shocks ε_{jt}^0 and ε_{jt}^1, but not the draws of it's rival j'. Hence, the output of the rival is a random variable. The expectation $E\left[rev_{ij}\right]$ above is taken over the rival's production decision $x_{j't}$ which, in turn, affects the market rent for the period, which, in turn, affect the sale price of a ship.

The discount factor for the two firms is given by β, the same one in the pricing equation.

Ships depreciate at rate $\delta \in (0,1)$ from period to period. That is

$$q_{t+1}^{\circ} = (1-\delta)\hat{q}_t.$$

(1) Define a Markov-perfect equilibrium of the above model.

(2) If you had time to derive analytical results, or time to program numerical examples of this model on the computer, discuss some properties of the equilibrium would would expect to find. What would happen if κ_1 is decreased while κ_2 is held fixed?

(3) Hold fixed the initial state q_0° at time zero and suppose a production subsidy for firm one is introduced at time $t=0$, so that $\kappa_1^{'} = \kappa_1 - \sigma_1$, where σ_1 is the subsidy. Contrast two cases. In the first, the subsidy to firm one applies only to production at time 0. In the second, the subsidy is permanent. Contrast the impact on the price of a ship of the two policies.

(4) In Barwick, Kalouptsidi, Zahur (ReStud forthcoming), the model specifies a static demand curve for ships to evaluate industrial policies. Suppose you were to use the lens of this paper to view data generated by the above model. Comment on whether you think the paper's approach would work well here or do you see any limitations?

Question 3

Consider two papers estimating the degree of market power in a particular settings:

- Paul L E Grieco, Charles Murry, Ali Yurukoglu, The Evolution of Market Power in the U.S. Automobile Industry, The Quarterly Journal of Economics, 2023
- Berger, David, Kyle Herkenhoff, and Simon Mongey. 2022. "Labor Market Power." American Economic Review, 112 (4): 1147-93.

Explain the two approaches, including the source of identification. Discuss advantages and disadvantages of each approach.

Question 4

Consider the following dynamic model of an industry. Periods are indexed by $t \in \{0, 1, 2, .., T\}$. There are N players, indexed by $i \in \{1, 2, .., N\}$. In each period t, firm i can either *produce* in the industry (indicated by $x_{it} = 1$), or *not produce* in the industry ($x_{it} = 0$).

Firm profit within a period has a deterministic component and a random component. The deterministic component of profit at time t depends upon a firm's own accumulated production as well as the rest of the industry's production prior to time t. Formally, for firm i define *own* and *rest-of-industry* accumulated prior production as

$$\begin{aligned} z_{it}^{own} &= \sum_{t'=0}^{t-1} x_{it'}. \\ z_{it}^{rest} &= \sum_{t'=0}^{t-1} \sum_{i' \neq i} x_{i't'} \end{aligned}$$

The deterministic component of firm-level profit at time t equals $f(z_{it}^{own}) + g(z_{it}^{rest})$, where f is a strictly *increasing* function satisfying

$$\begin{aligned} f(0) &= 0\,, \\ \lim_{z \to \infty} f(z^{own}) &= \phi, \text{ where } \phi > 0 \\ f'(z^{own}) &> 0, \end{aligned}$$

and g is a strictly *decreasing* function satisfying

$$\begin{aligned} g(0) &= \gamma, \text{ where } \gamma > 0 \\ \lim_{z \to \infty} g(z^{rest}) &= 0, \\ g'(z^{rest}) &< 0. \end{aligned}$$

The random component of profit depends on whether or not the firm produces. Let ε_{it}^0 be the random profit for firm i at time t to not produce and let ε_{it}^1 be the random profit from producing. Assume this is drawn i.i.d. across firms and over time from the standardized type 1 extreme value distribution.

The entry decisions of the N players are made simultaneously in each period t. Firm i, when making its entry decision in period t, observes z_t and and its own profit shocks ε_{it}^0 and ε_{it}^0, but not $\varepsilon_{i't}^0$ and $\varepsilon_{i',t}^1$ for $i' \neq i$. That is, a firm's own profit shock is private information.

Firms discount future payoffs with discount factor $\beta < 1$.

(a) Suppose $T = \infty$ (i.e. that the horizon is infinite). Define a Markov-perfect equilibrium in this model. Calculate the limiting probability a firm chooses to produce as $t \to \infty$.

(b) Suppose that at $t = 0$, $z_{i0}^{own} = 0$ and $z_{i0}^{rest} = 0$ all i. Suppose $\phi = \gamma$. Continue to assume $T = \infty$. Compare the limiting probability a firm chooses to produce as $t \to \infty$, with the probability a firm choose to product at $t = 0$. In your analysis, compare the case of $\beta = 0$ with $\beta > 0$.

(c) Discuss the issue of uniqueness of an Markov perfect equilibrium. Suppose things were flipped, so that f is a strictly decreasing function while g is a strictly increasing function. Discuss how this might impact the question of whether equilibrium is unique. In your analysis, begin with the assumption of $\beta = 0$ and state and prove a formal result. Then explain how things potentially differ when $\beta > 0$, but for this case you do not have to prove a formal result.

(d) Suppose this model is simulated on a computer, on multiple sample paths, and you observe the entry and exit decisions of each firm on each sample path. Discuss strategies for estimating the functions $f(z^{own})$ and $g(z^{rest})$. Discuss what the potential different patterns in the data that would enable you to separately identify ϕ and γ.

(e) Consider a policy to set a subsidy σ for production in each period. Discuss how you would use the estimated model to evaluate the welfare effects of this policy.