MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2020**

FINAL EXAMINATION

Answer ***two*** of the following three questions.

1. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \gamma \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \gamma \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $\gamma = 2$ and $(w_1, w_2) = (4, 6)$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that, rather than endowments of consumption goods, the consumers have endowments of labor $(\bar{\ell}_t^t, \bar{\ell}_{t+1}^t) = (\bar{\ell}_1, \bar{\ell}_2)$ and $\bar{\ell}_1^0 = \bar{\ell}_2$. The representative consumer in the initial old generation has an endowment of capital $\bar{k}_1^0$ and an endowment m of fiat money. Final output, which can be consumed or invested is produced using the production function $\theta k_t^\alpha \ell_t^{1-\alpha}$, $\theta > 0$, $0 < \alpha < 1$, and a fraction δ, $0 \le \delta \le 1$, of capital depreciates every period. Define a sequential markets equilibrium for this economy.

2. Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t c_t^{\rho} .$$

Here $0 < \beta < 1$ and $0 < \rho < 1$. The consumer is endowed with 1 unit of labor in each period and with $\bar{k}_0$ units of capital in period 0. Feasible allocations satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq \theta k_t^{\alpha} \ell_t^{1-\alpha}$$
$$c_t, k_t \geq 0 ,\ 1 \geq \ell_t \geq 0 .$$

Here $\theta > 0$, $0 < \alpha < 1$, and $0 \leq \delta \leq 1$.

(a) Formulate the problem of maximizing the representative consumer's utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman equation.

(b) Let $K = [0, \tilde{k}]$. Explain how you can use the feasibility condition to choose $\tilde{k}$ to be the maximum sustainable capital stock. Let $C(K)$ be the space of continuous bounded functions on K. Endow $C(K)$ with the topology induced by the sup norm

$$d(V,W) = \sup_{k \in K} |V(k) - W(k)| \text{ for any } V, W \in C(K) .$$

Define a contraction mapping $T : C(K) \to C(K)$.

(c) State Blackwell's sufficient conditions for T to be a contraction. (You do not need to prove that these conditions are sufficient for T to be a contraction.)

(d) Using the Bellman equation from part a, define the mapping for the value function iteration algorithm,

$$V_{n+1} = T(V_n) ,$$

where $T : C(K) \to C(K)$; that is $V = T(V)$ is the Bellman equation. (You do not need to prove that $T(V) \in C(K)$ for all $V \in C(K)$.) Prove that T satisfies Blackwell's sufficient conditions to be a contraction.

(e) Specify an economic environment for which the solution to the social planner's problem in part a is a Pareto efficient allocation/production plan. Define a sequential markets equilibrium for this environment. Explain how you could use the value function iteration algorithm $V_{n+1} = T(V_n)$ to calculate the unique sequential markets equilibrium. (You do not have to prove that this equilibrium is unique.)

3. Consider an economy with a representative, infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. The consumer owns one unit of labor in each period and $\bar{k}_0$ units of capital in period 0. Suppose that feasible consumption/investment plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \le \theta k_t^{\alpha} \ell_t^{1-\alpha}.$$

(a) Suppose that the consumer borrows b_{t+1} bonds in period t to be paid off in period $t+1$. The consumer's initial endowment of bonds is $\bar{b}_0 = 0$, the wage rate in period t is w_t, the rental rate on capital is r_t^k, and the interest rate on bonds is r_t^b. Write down the consumer's utility maximization problem in a sequential markets economy. Explain why you need to include a constraint to rule out Ponzi schemes. Write down the Euler conditions and the transversality conditions for this problem. Define a sequential markets equilibrium with borrowing and lending for this economy. Prove that in equilibrium $r_t^k - \delta = r_t^b$ if $k_t > 0$.

(b) Define the Arrow-Debreu equilibrium for this economy.

(c) Carefully state theorems that relate the equilibrium allocations in parts a and b.

(d) Suppose that now there are two types of consumers. The representative consumer of type 1 has the endowment of labor $(\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, \ldots) = (2,1,2,1,\ldots)$ and the endowment of capital $\bar{k}_0^1$. The representative consumer of type 2 has the endowment of labor $(\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, \ldots) = (1,2,1,2,\ldots)$ and the endowment of capital $\bar{k}_0^2$. Define a sequential markets equilibrium for this economy.

(e) Does the equilibrium allocation/production plan in part d solve a dynamic programming problem? If it does, write down the Bellman's equation for this problem. If it does not, explain carefully why it does not.