

MIDTERM EXAMINATION

Answer **two** of the following three questions..

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer  $i$ ,  $i = 1, 2$ , has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i.$$

Here  $\beta$ ,  $0 < \beta < 1$ , is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, \dots) = (2, 1, 2, 1, \dots)$$

$$(w_0^2, w_1^2, w_2^2, w_3^2, \dots) = (1, 4, 1, 4, \dots).$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique, but you do not have to prove this fact.)

(d) Suppose now that there is a production technology that transforms labor and capital into output that can be consumed or saved as capital:

$$y_t = \theta k_t^\alpha \ell_t^{1-\alpha},$$

where  $\theta > 0$  and  $1 > \alpha > 0$ . Capital depreciates at the rate  $\delta$ ,  $1 > \delta > 0$ , every period. The consumers' endowments of labor are

$$(\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, \dots) = (2, 1, 2, 1, \dots)$$

$$(\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, \dots) = (1, 4, 1, 4, \dots).$$

Their endowments of capital in period 0 are  $\bar{k}_0^i > 0$ ,  $i = 1, 2$ . Define a sequential markets equilibrium for this economy.

2. Consider an overlapping generations economy in which the representative consumer born in period  $t$ ,  $t = 1, 2, \dots$ , has the utility function over consumption of the single good in periods  $t$  and  $t + 1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t$$

and endowments  $(w_t^t, w_{t+1}^t) = (w_1, w_2)$ . Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment  $w_1^0 = w_2$  of the good in period 1 and endowment  $m$  of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that  $m = 0$ . Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Still supposing that  $m = 0$ , relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period  $t$ ,  $t = 0, 1, \dots$ , can be transformed into  $\theta > 0$  units of the good in period  $t + 1$ . Define a sequential markets equilibrium for this economy. Provide a condition on  $(w_1, w_2)$  that guarantees that the storage technology will be used in equilibrium.

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where  $0 < \beta < 1$ . This consumer has an endowment of  $\bar{\ell} = 1$  units of labor in each period and  $\bar{k}_0$  units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq \theta k_t^\alpha \ell_t^{1-\alpha}.$$

- (a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.
- (b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.
- (c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.
- (d) Suppose now that the representative consumer faces the choice of selling his labor services or consuming them as leisure. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t (\gamma \log c_t + (1-\gamma) \log x_t)$$

where  $x_t = 1 - \ell_t$  is leisure. Define an Arrow-Debreu equilibrium for this economy.

- (e) Define sequential market equilibrium for the economy in part d.