MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2020**

PROBLEM SET #1

1. Consider an economy two infinitely lived consumers, each of whom has the same utility function,

$$u(c_0^i, c_1^i, \ldots) = \sum_{t=0}^{\infty} \beta^t \log\ c_t^i ,$$

where $0 < \beta < 1$. Suppose that consumer 1 has the endowments

$$(w_0^1, w_1^1, w_2^1, w_3^1, \ldots) = (6, 2, 6, 2, \ldots)$$

and consumer 2 has the endowments

$$(w_0^2, w_1^2, w_2^2, w_3^2, \ldots) = (2, 6, 2, 6, \ldots)$$

a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

b) Define a Pareto efficient allocation for this economy. Calculate a Pareto efficient allocation by maximizing a weighted sum of utilities, $\alpha_1 u_1 + \alpha_2 u_2$.

c) Define an Arrow-Debreu equilibrium with transfers. Find the transfer payments necessary to implement the Pareto efficient allocation in part b as equilibrium with transfers. Demonstrate that the transfer payments are homogeneous of degree one in (α_1, α_2) and sum to 0.

d) Find the transfer payments necessary to implement the allocation $(c_t^1, c_t^2) = (4, 4)$ as an equilibrium with transfers.

e) Calculate the (unique) Arrow-Debreu equilibrium of this economy.

f) Define a sequential markets equilibrium. Calculate the unique sequential markets equilibrium of the economy.

g) Redo the calculations in part e for the economy in which

$$(w_0^1, w_1^1, w_2^1, w_3^1, \ldots) = (6, 2, 6, 2, \ldots)$$

$$(w_0^2, w_1^2, w_2^2, w_3^2, \ldots) = (4, 4, 4, 4, \ldots).$$

2. Consider a simple overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function

$$u(c_t^t, c_{t+1}^t) = c_t^t + \left[\left(c_{t+1}^t\right)^b - 1\right]/b,$$

where $b < 1$. Suppose that his endowment is $(w_t^t, w_{t+1}^t) = (w_1, w_2)$.

a) What is the utility function in the case where $b = 0$? [Hint: use l'Hôpital's rule.]

b) Write down the utility maximization problem in an environment with Arrow-Debreu markets. Derive the excess demand functions $y(p_t, p_{t+1})$ and $z(p_t, p_{t+1})$. Demonstrate that they are homogeneous of degree zero and that they satisfy Walras's law.

c) Suppose that the representative consumer in the first generation has the utility function $u^0(c_1^0) = \left[\left(c_1^0\right)^b - 1\right]/b$. This consumer is endowed with $w_1^0 = w_2$ of the good in period 1 as well as m units of fiat money, where m can be positive, negative, or 0. Explain the role of m. Define an Arrow-Debreu equilibrium of this model. Write down the equilibrium conditions using the excess demand functions.

d) Find an expression for the offer curve for this model. (Hint: you have to solve for y as a function of z.)

e) Suppose that $w_1 = 2$ and $w_2 = 0.75$. Draw the offer curve for the three cases $b = 0.5$, $b = 0$, and $b = -1$.

f) Define a sequential market equilibrium for this economy. Suppose that you have calculated the Arrow-Debreu equilibrium in part d. Explain how you can use the Arrow-Debreu equilibrium to calculate the sequential market equilibrium.